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Double Q^2-rescaling model and the nuclear effect of the parton distribution functions*

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Abstract. In order to overcome the shortcoming of nonconservation of nuclear momentum existing in the original Q^2-rescaling model (OQ^2RM) and avoid introducing nuclear shadowing factor, we proposed a double Q^2-rescaling model (DQ^2RM) for the parton distributions of the bound nucleon. Using the experimental data of lepton-nucleus deep inelastic scattering (DIS) and the condition of the nuclear momentum conservation, the Q^2-rescaling parameters of various partons for Sn, Fe, Ca and C nuclei are determined. The rescaling parameters of valence quark distributions are larger than unity and gradually increase with atomic number A, on the contrary, the rescaling parameters of sea quark distributions and gluon distributions are smaller than unity, and slowly decrease with A. By using this model, the experimental data of the DIS process, the nuclear Drell-Yan process and J/ψ photoproduction process are consistently and quite satisfactorily explained.

1 Introduction

In 1982, the European Muon Collaboration at CERN discovered in the muon-nucleus deep inelastic scattering (DIS) that the structure function F_2 of bound nucleon is quite different from that of free nucleon within the region $0.1 \leq x \leq 0.7$ [1], this nuclear effect was thus called the EMC effect. Since the discovery of the EMC effect, physicists have proposed several models and made out more or less satisfactory explanation for it [2]. But, for the shadowing effect emerged within very small x region in DIS process [3] and the A-dependence of the differential cross section of h-A Drell-Yan process [4], only the constituent quark model, Q^2-rescaling model [5] and the extended x-rescaling model [6], after introducing the nuclear shadowing factor, can make out satisfactory explanation. In addition, only the three models mentioned above have made out quantitative description for J/ψ photoproduction on the nuclear target.

In the Q^2-rescaling model with shadowing factor, except for the parameter relative to shadowing, for the distribution functions of valence quark, sea quark and gluon within the bound nucleon, a common Q^2-rescaling parameters $\xi(A)$ was employed. Although the model can explain several kinds experimental processes mentioned above, but as pointed by Li Guanglie et al. [6] it leads to non-conservation of nuclear momentum, that is, the momentum of a nucleus is no longer equal to the sum of momenta of valence quarks, sea quarks and gluons including in the nucleus. They thought that for the valence quark distribution and sea quark distribution, two different Q^2-rescaling parameters should be employed.

As a phenomenological approach to investigate the parton distribution functions of a bound nucleon, here we presented a model, in which three Q^2-rescaling parameters were employed for the distributions of valence quark, sea quark and gluon respectively, and the artificial shadowing factor was not used. Due to restriction of nuclear momentum conservation, only two among the three parameters are independent, so we call this model the double Q^2-rescaling model (DQ^2RM).

From large amounts of available experimental data, it can be seen that for the both of free nucleon and bound nucleon, the x distribution and its evolution with Q^2 of valence quarks are quite different from those of sea quarks and gluons, The distribution of valence quarks is mainly restricted within the region of $x > 0.1$, the "hardness" of the distribution allows one to imagine that the valence quarks are confined within a bag with a distinct boundary and the behavior of its evolution is described by GLAP equation [7]. The EMC effect mentioned above mainly reflects the change of valence quark distribution of bound nucleon as compared with free nucleon. For the distribu-

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tions of sea quark and gluon, however, the case is much complex. The distribution of sea quark and gluon mainly restricted within the region of $x < 0.1$, and quickly increases with x decreasing. The case is more obvious for the gluon distribution. Therefore, they play dominant role in the region of very small x [8]. As well known, the longitudinal size of parton with momentum fraction x is $\Delta z = \frac{1}{xp}$, p is the momentum of nucleon in a certain frame of reference with high speed, then usual picture that partons are confined in a colour bag becomes indistinct. In relation with the case, soft partons with very small x value from adjacent nucleons lying in the same longitudinal line within nucleus may penetrate and overlap each other. The smaller x value, and the larger the atomic number A, the more outstanding the phenomena. Then, recombination or fusion between two or several partons from different nucleons could occur. The partons with very small x recombine and form into a parton with slightly larger x, and this process naturally leads to the shadowing in very small x region and antishadowing in slightly larger x region, thereby, after taking into account the recombination, the artificial shadowing factor is not needed.

Based on above mentioned understanding, we present DQ^2RM. Using this model, the influence of nuclear environment on the parton distributions and the experimental data on l-A DIS process, h-A Drell-Yan process and γ^*-A J/ψ production can be consistently and satisfactorily explained. In contrast with the rescaling parameter $\xi_V(A)$ of valence quark distribution is larger than unity and slowly increases with A increasing, a characteristic feature of our model is that both the rescaling parameters $\xi_S(A)$ of sea quark distribution and $\xi_G(A)$ of gluon distribution are smaller than unity and slowly decrease with A increasing. This feature just right meets the needs of the Q^2 evolution picture of parton distributions in small x region of bound nucleon and nuclear momentum conservation. We shall present experimental support for this feature in the Sect. 3.

2 The double Q^2-rescaling model and relative formulas

For a nucleus A, our model assumes three different Q^2-rescaling parameters for the distributions of valence quarks, sea quarks and gluons in the bound nucleon respectively:

$$\begin{aligned} q^A_{Vi}(x,Q^2) &= q^N_{Vi}(x,\xi^A_V Q^2)\,,\\ q^A_{Si}(x,Q^2) &= q^N_{Si}(x,\xi^A_S Q^2)\,,\\ \bar{q}^A_{Si}(x,Q^2) &= \bar{q}^N_{Si}(x,\xi^A_S Q^2)\,,\\ G^A(x,Q^2) &= G^N(x,\xi^A_G Q^2)\,. \end{aligned} \tag{1}$$

where i denotes the flavor of quark. That is to say, the distributions of various partons of bound nucleon may be obtained by Q^2 rescaling for the distributions of corresponding partons of free nucleon. For all of nuclei and any Q^2 values, the momentum conservation condition

$$\int_0^1 x dx \{\sum_i [q^A_{Vi}(x,Q^2) + q^A_{Si}(x,Q^2) + \bar{q}^A_{Si}(x,Q^2)] + G^A(x,Q^2)\} = 1\,. \tag{2}$$

should be respected. Because of the restriction of formula (2), only two among three Q^2-rescaling parameters are independent, so this model is called the double Q^2-rescaling model.

In the DQ^2RM, the structure function and gluon distribution function of bound nucleon respectively is

$$F^A_2(x,Q^2) = \sum_i e_i^2 x[q^N_{Vi}(x,\xi^A_V Q^2) + q^N_{Si}(x,\xi^A_S Q^2) + \bar{q}^N_{Si}(x,\xi^A_S Q^2)] \tag{3}$$

and

$$xG^A(x,Q^2) = xG^N(x,\xi^A_G Q^2)\,, \tag{4}$$

$F^A_2(x,Q^2)$ denotes the average nucleon structure function of a ideal nucleus with the equal number of protons and neutrons ($N = Z = \frac{1}{2}A$).

The nuclear effect of the nucleon structure function and gluon distribution may be represented by following ratios:

$$R^{A/D}(x,Q^2) = \frac{F^A_2(x,Q^2)}{F^D_2(x,Q^2)}\,, \tag{5}$$

$$R^{A/D}_G(x,Q^2) = \frac{G^A(x,Q^2)}{G^D(x,Q^2)}\,, \tag{6}$$

usually, these ratios calculated by a certain theoretical model are used to compare with the experimental data.

For the Drell-Yan process in p-A collision, the nuclear effect is represented by the ratio

$$T^{A/N}(x_t,Q^2) = \frac{\int \frac{d^2\sigma^{p-A}(x,x_t,Q^2)}{dxdx_t}dx}{\int \frac{d^2\sigma^{p-N}(x,x_t,Q^2)}{dxdx_t}dx}\,, \tag{7}$$

where

$$\frac{d^2\sigma^{p-A}(x,x_t,Q^2)}{dxdx_t} = \frac{1}{3}\frac{4\pi\alpha^2}{3M^2}\sum_i e_i^2\{[q^p_{Vi}(x,Q^2) + q^p_{Si}(x,Q^2)]\bar{q}^A_{Si}(x_t,Q^2) + \bar{q}^p_{Si}(x,Q^2)[q^A_{Vi}(x_t,Q^2) + q^A_{Si}(x_t,Q^2)]\}\,, \tag{8}$$

$$\frac{d^2\sigma^{p-N}(x,x_t,Q^2)}{dxdx_t} = \frac{1}{3}\frac{4\pi\alpha^2}{3M^2}\sum_i e_i^2\{[q^p_{Vi}(x,Q^2) + q^p_{Si}(x,Q^2)]\bar{q}^N_{Si}(x_t,Q^2) + \bar{q}^p_{Si}(x,Q^2)[q^N_{Vi}(x_t,Q^2) + q^N_{Si}(x_t,Q^2)]\}\,, \tag{9}$$

where M^2 denotes the invarint mass of lepton-pair($\mu\bar{\mu}$) produced in the process.

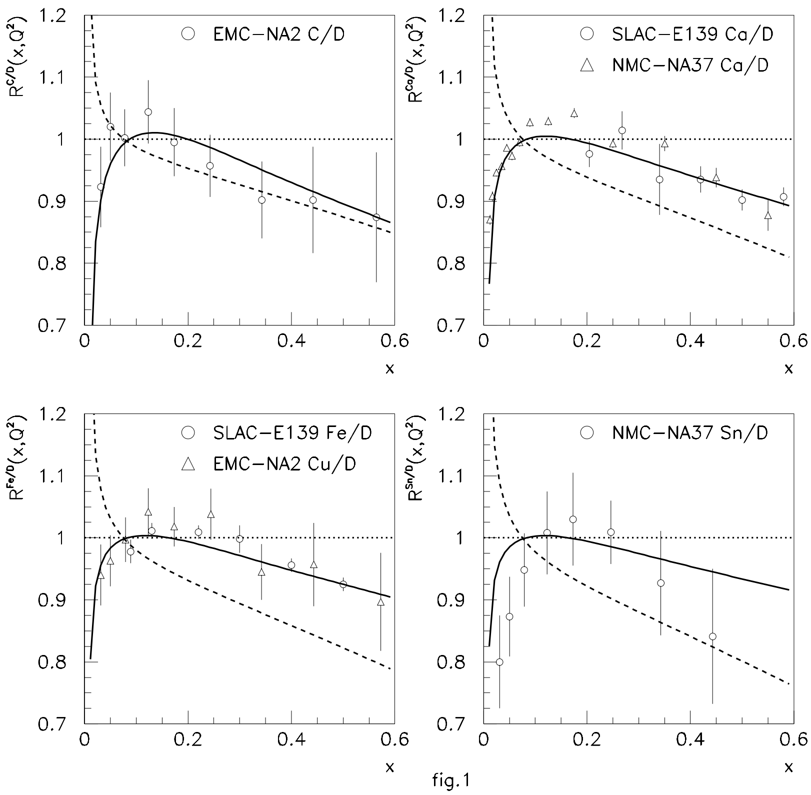

Fig. 1. The ratio $R^{A/D}(x,Q^2)$ for nuclei C^{12}, Ca^{40}, Fe^{56}, Sn^{119} versus the momentum fraction x within the Q^2 range given in [3], where corresponding experimental data [3] are shown. The *solid lines* are the results of our model, the *dashed lines* are the results of the old Q^2−rescaling model (In the OQRM, the Q^2-rescaling parameters ξ for nuclei C^{12}, Ca^{40}, Fe^{56}, Sn^{119} are 1.6, 1.86, 2.02, 2.24, respectively)

Three Q^2-rescaling parameters ξ_V, ξ_S and ξ_G are determined by the following method: firstly, we choose suitable ξ_V and ξ_S to explain the nuclear effect in l-A DIS process; secondly, we describe p-A Drell-Yan process with the same values of ξ_V and ξ_S to test them, and finally determine ξ_G using nuclear momentum conservation and explain the nuclear effect of J/ψ photoproduction process at nuclear target.

For the parton distribution function of free nucleon, we adopt the parton momentum distributions given by M. Gluck, E. Reya and A. Vogt (GRV) [9], because this formalism allows to be evoluted inversely to $Q^2 \approx 1.0\,\mathrm{GeV}^2$. This is quite important point for getting the distribution functions of sea quark and gluon of bound nucleon by formula (1) at $\xi_S^A < 1$ and $\xi_G^A < 1$.

At present, the statistics and the precision of experimental data on l-A DIS process, h-A Drell-Yan process and γ^*-A J/ψ production process is insufficient to test very precise theoretical calculation, so that in our calculation, only the leading order approximation on the formulas of hard subprocesses and parton distribution functions are considered.

The Q^2-rescaling parameters ξ_V^A, ξ_S^A and ξ_G^A on nuclei C^{12}, Ca^{40}, Fe^{56} and Sn^{119} obtained by above method from the data of references [3, 4, 10] are listed in Table 1. It is worth to note that, all of ξ_V^A are larger than unity and increase with A increasing, but all of ξ_S^A and ξ_G^A are smaller than unity and decrease with A increasing. In the next section, we shall present a experimental evidence supporting this result.

Table 1.

	ξ_V	ξ_S	ξ_G
C	1.3	0.7	0.86
Ca	1.35	0.67	0.81
Fe	1.41	0.62	0.76
Sn	1.57	0.45	0.58

3 Results and discussion

Using the values of Q^2-rescaling parameters listed in Table 1 and formulas (1), (3) and (4), in which LO parton momentum distributions of free nucleon given by GRV are adopted [9], we obtained various parton distribution functions and average structure function F_2 of bound nucleon in nuclei C^{12}, Ca^{40}, Fe^{56} and Sn^{119}. Furthermore, using formulas (5)–(9), the theoretical predictions on $R^{A/D}$ (x,Q^2), $T^{A/N}(x_t,Q^2)$ and $R_G^{A/D}(x,Q^2)$ by the double Q^2-rescaling model are calculated.

Theortical curves of $R^{A/D}(x,Q^2)$ on l-A DIS processes for nuclei C^{12}, Ca^{40}, Fe^{56}, Sn^{119} within the Q^2 range given in reference [3] are separately plotted in Fig. 1, where corresponding experimental data [3] are shown. It can be seen that by our model, only taking different parameters for valence quark distribution and sea quark distribution, without introducing the the shadowing factor, the experimental data of nuclear effect in l-A DIS process can be explained quite well in almost the whole x region.

Theoretical curves of $T^{A/D}(x_t,Q^2)$ on p-A Drell-Yan process for nuclei C^{12}, Ca^{40} and Fe^{56} within region of $0.025 \le x_t \le 0.30$ and $4 \le M_{l\bar{l}} \le 9\,\mathrm{GeV}$ and at $E_{CM} =$

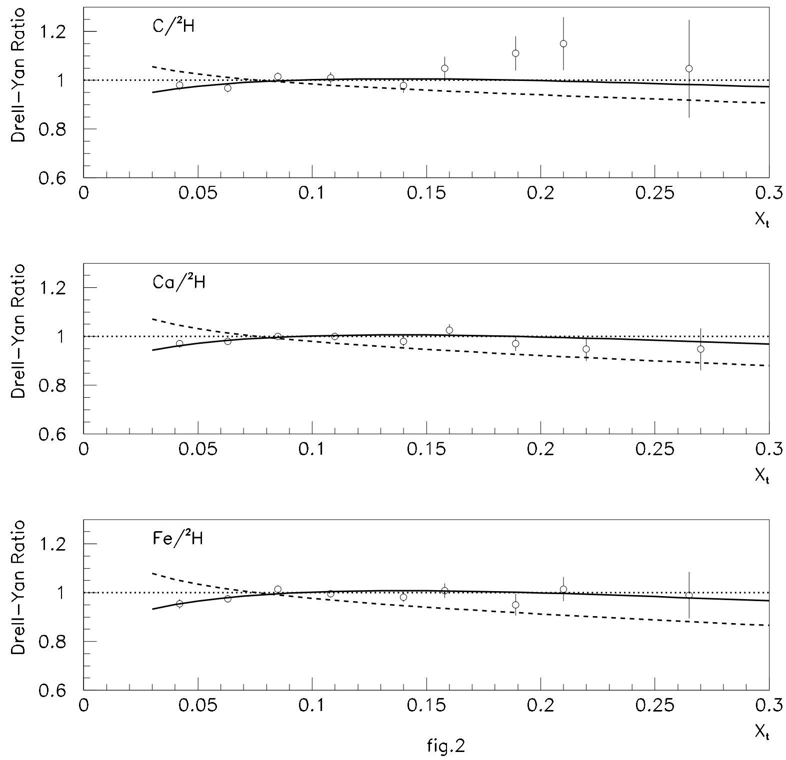

Fig. 2. The ratio $T^{A/D}(x_t, Q^2)$ for nuclei C^{12}, Ca^{40} and Fe^{56} versus x within region of $0.025 \leq x_t \leq 0.30$ and $4 \leq M_{l\bar{l}} \leq 9\,\mathrm{GeV}$ and at $E_{CM} = 40\,\mathrm{GeV}$, where corresponding experimental data [4] are shown. The meaning of lines is the same as that in Fig. 1

40 GeV are separately plotted in Fig. 2a–c, where corresponding experimental data [4] are shown. We can see that our results are in qualitative agreement with the data, and especially in the region of $x_t \leq 0.15$, the agreement is quite satisfactory. In addition, within the region $0.15 \leq x_t \leq 0.30$, the agreement case for heavier nucleus is clearly better than for lighter one. Furthermore comparing our results which the prediction given by original Q^2-rescaling model, it can be seen that the DQ^2RM overcame the shortcoming that the tendency of the curve given by original Q^2-rescaling model is wholly different from the data.

Theoretical curve of $R_G^{Sn/C}(x, Q^2)$ (the ratio of gluon distribution function of nucleus Sn to one of nucleus C) is plotted in Fig. 3, where the experimental data [10] of the ratio measured in γ^*+A→ J/ψ+X process are shown, the result calculated by original Q^2-rescaling model is also plotted in the Fig. 3. Although there are quite large errors in the experimental data, it can still be seen that the prediction given by our model is clearly better than OQ^2RM.

Curves of $R_G^{A/D}(x, Q^2)$ from DQ^2RM within region $0.1 \leq x \leq 0.2$ for nuclei C, Ca, Fe and Sn are plotted in Fig. 4. It can be seen that slope of these curves slowly increasing with A increasing. This case reflects that the influence of gluon recombination is enhanced with A increasing.

In the following paragraphs, we shall discuss the problem of the rescaling parameters $\xi_V > 1$, $\xi_S < 1$ and $\xi_G < 1$.

From above comparision of the calculating results obtained by our model with the experimental data we can see that our model has certain rationality, and specific values of ξ_V, ξ_S and ξ_G listed in Table 1 are supported by the relevant experiments.

Firstly, in our model, $\xi_V > 1$. This is the same as the OQ^2RM. The latter model can well explain the nuclear effect of the DIS process in the region $0.2 < x < 0.7$, where the momentum distribution of nucleus is mainly determined by valence quark, so the value of ξ_V in our model should roughly be consistent with the latter. In physics, $\xi_V > 1$ can be explained by the swelling of nucleon. When the nucleon is swollen, according to uncertainty principle, the momentum distribution of valence quarks moves to small x region, i.e. the momentum distribution of valence quarks was softened.

Secondly, because of the following reasons, we assume that $\xi_S < 1$. The first reason is that the OQ^2RM can not explain the experimental data of nuclear effect of the l-A DIS process and nuclear Drell-Yan process in small x region, where the nuclear effect is mainly determined by the change of sea quark distribution, so we need to reconsider the Q^2-rescaling parameter of sea quark. The second reason is that some authors of this paper calculated the values of the nuclear effect function $R_S^A(x, Q^2)$ of sea quark distribution by using the experimental data of l-A DIS process and nuclear Drell-Yan process [11] and found that in region $x \leq 0.2$ $R_S^A(x, Q^2)$ increases from the value smaller than unity to the value slightly larger than unity with x increasing, and the larger the number A, the smaller the value of $R_S^A(x, Q^2)$. If we use Q^2-rescaling model to describe this change of sea quark distribution influenced by nuclear environment, it must demand $\xi_S < 1$, and the larger the A, the smaller the ξ_S.

Thirdly, we shall present another experimental surport on $\xi_G < 1$. It is well known that the gluon distribution

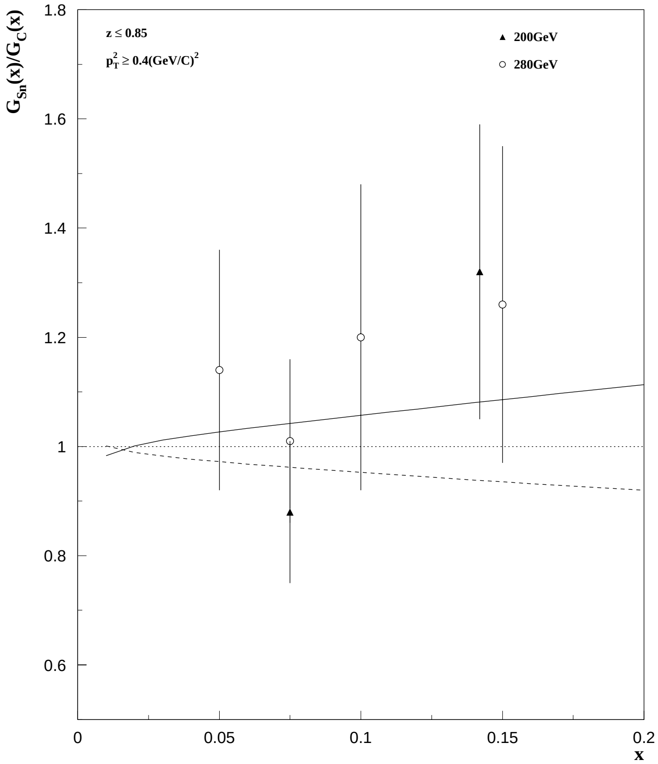

Fig. 3. The ratio $R_G^{Sn/C}(x, Q^2)$ versus x, where the experimental data [10] are shown. The meaning of lines is the same as that in Fig. 1

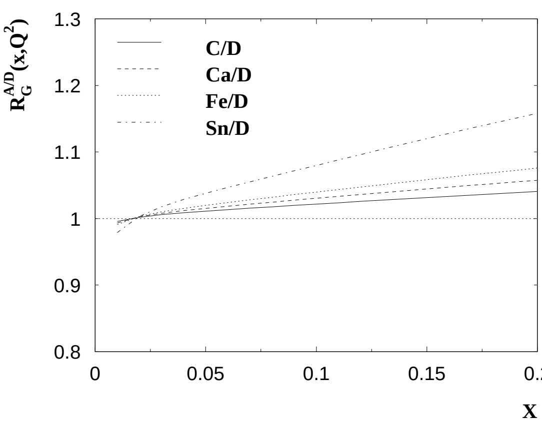

Fig. 4. The ratio $R_G^{A/D}(x, Q^2)$ from DQ^2RM versus x within region $0.1 \leq x \leq 0.2$ for nuclei C^{12}, Ca^{40}, Fe^{56} and Sn^{119}

of nucleon is the most important in the small x region, and in the region of $x < 10^{-2}$, gluon distribution almost saturates the parton distributions of nucleon. In the case, structure function $F_2(x, Q^2)$ is fully determined by the gluon distribution, and it can be obtained from PQCD [12]

$$F_2(x, Q^2) = \sum_i e_i^2 \int_x^1 dz \int^{Q^2} \frac{dk^2}{k^2} P_{q_iG}(\frac{x}{z}) \frac{\alpha_s(k^2)}{4\pi} \frac{\partial[zG(z, k^2)]}{\partial(\ln k^2)}$$

$$\approx \sum_i e_i^2 \frac{\alpha_s(Q^2)}{4\pi} \int_x^1 P_{q_iG}(\frac{x}{z}) zG(z, Q^2) dz \,, \qquad (10)$$

where $G \to q_i$ splitting function $P_{q_iG}(x) = \frac{1}{2}[x^2 + (1-x)^2]$. Due to the small the x value, the more important the gluon distribution, therefore equation (10) can be approximately expressed as

$$F_2(x, Q^2) \sim \frac{1}{8\pi} \sum_i e_i^2 \alpha_s(Q^2) G(x, Q^2) \Delta x. \qquad (11)$$

Recently, NA37/NMC collaboration at CERN measured the ratio F_2^{Sn}/F_2^C in the region of $0.0125 \leq x \leq 0.55$ and $1 \leq Q^2 \leq 10^2\,\mathrm{GeV}^2$ [13]. Their result shows that in the region of $x < 3 \times 10^{-2}$, $F_2^{Sn}(x, Q^2)/F_2^C(x, Q^2)$ is clearly less than unity, and this result means $G^{Sn}(x, Q^2) < G^C(x, Q^2)$ in the region. If we use Q^2-rescaling approach to get $G^{Sn}(x, Q^2)$ from $G^C(x, Q^2)$ and assume

$$G^{Sn}(x, Q^2) = G^C(x, \xi^{Sn/C} Q^2) \,, \qquad (12)$$

where $\xi^{Sn/C}$ should be smaller than unity, that is to say, the gluon distribution function of a heavier nucleus may be obtained from one of a slighter nucleus by Q^2 rescaling approach and using Q^2 rescaling parameter smaller than unity. Therefore, we may conjecture that the gluon distribution functions of all nuclei may be obtained from one of free nucleon or deuterium by this approach and using ξ_G^A smaller than unity, moreover, the values of ξ_G^A slowly decrease with atomic number A increasing.

In summary, we presented DQ^2RM, in which different rescaling parameters ξ_V, ξ_S and ξ_G are assumed, and the shadowing factor is not introduced. By this model, the experimental data of the l-A DIS process, the nuclear Drell-Yan process and $\gamma^* - A$ J/ψ production process were consistently and quite satisfactorily explained. But, this model is a initial and phenomenological model, on the one hand, it needs to be further improved as more precise experimental data presented, on the other hand, its physical foundation still needs to be explored more elaborately.

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